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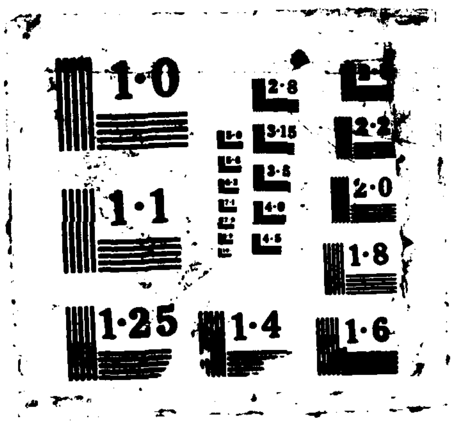
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The Application of Doubly-Truncated Hydrometeor Distributions to Numerical Cloud Models

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9 February 1987

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ATMOSPHERIC SCIENCES DIVISION PROJECT 6670
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"This technical report has been reviewed and is approved for publication"

FOR THE COMMANDER

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A number of meteorological applications involving clouds require being able to determine a reasonable hydrometeor particle size distribution when given the mean hydrometeor mass density ("liquid-" or "ice-water content") over a region of space. Such applications include numerical cloud models with "parameterized" or "bulk" microphysics, which have often assumed exponential size distributions. The present study proposes a parameterization scheme based on a doubly-truncated exponential particle size distribution, i.e. a distribution that is truncated at both the large and small diameter ends. The necessary inputs to the scheme are the liquid or ice water content and the temperature of the sample, and, in some circumstances, an estimate of the lower truncation limit. Outputs include the parameters of the exponential distribution, including the upper truncation limit. A special set of relationships is used for rain when the largest particles exceed the assumed breakup diameter of 6 mm.

The scheme relies on theoretical relationships derived from the equation for the double-truncated distribution, and on empirically derived relationships. The empirical expressions

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include relationship between M and Z (where M is the liquid or ice water content and Z is the radar reflectivity) and values for the product ΛD_m (where Λ is the parameter of the exponential distribution and D_m is the upper truncation limit). In an earlier study ΛD_m was found to be nearly-constant at 8.2 for rain, and to be a slowly-varying function of temperature for ice hydrometeors.

This paper presents a simple, analytical expression for Λ, derived by assuming the lower truncation limit to be zero. With an independent estimate of the value of the lower truncation limit, a factor f is also determined, which will correct Λ to 99 percent of its actual value.

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1. INTRODUCTION

The presence of cloud or precipitation particles or "hydrometeors" (both liquid and frozen) has a significant impact on many Air Force systems and operations. Because of this, it is often necessary to determine reasonable particle size distributions from limited data, which may include measurements of mean radar reflectivity or mean densities of liquid water or ice over a region of the atmosphere. An important tool for studying the evolution of cloud properties and processes is the numerical cloud model. When clouds are modeled in two or three dimensions, it is again often necessary to obtain size distributions of liquid water or ice particles given only data averaged over a region of space or "grid volume". The averaged data generally include the mixing ratios of the various hydrometeor species (cloud water, rain, cloud ice, snow or aggregates, graupel/hail, etc.) from which one can calculate the density ("liquid water content" or "ice water content") of each species. In the cloud model it is necessary to know the distribution of each particle type in order to derive mathematical expressions for the conversion processes among the various species (for example the rate of freezing of raindrops to form graupel depends on the size distribution of the raindrops). Once a suitable relationship

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between mean cloud properties and size distribution is derived and validated, it is also useful for including into diagnostic cloud modeling and artificial intelligence schemes involving clouds.

The Cloud Physics Branch of the Atmospheric Sciences Division at the Air Force Geophysics Laboratory (AFGL) has been utilizing a 3-dimensional, numerical-cloud model (hereafter referred to as the 3-D model) for predicting mixing ratios of hydrometeor liquid and ice water content from bulk microphysical parameterizations. This AFGL 3-D model is a version of that developed at Colorado State University.[1,2] The parameterizations rely upon assumed exponential size distributions for deriving the conversion processes occurring amongst the various hydrometeor species.

The assumption of exponential distributions, where the logarithmic values of the number of hydrometeors decrease as a function of increasing diameter size, is generally accepted as being reasonably descriptive and is supported by various studies.[3-7] Figure 1a shows a typical semi-logarithmic, number-density plot where Figure 1b, the same data plotted linearly, emphasizes the magnitude differences in numbers of hydrometeors at the small and large diameters.

In the current modeling, the assumed exponential distributions are generally defined by constant values of the total number of particles or for the slope and/or zero intercept of the semi-logarithmic number density plot, although observations do not support these generalities. Derived distributions employing these types of assumptions have very little chance of agreeing with measurements of actual events.

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1. Tripoli, G.J., and Cotton, W.R. (1982) The Colorado State University three-dimensional cloud/mesoscale model - Part I: General theoretical framework and sensitivity experiments, J. Rech. Atmos. 16:185-219.
2. Cotton, W.R., Tripoli, G.J., Rauber, R.M., and Mulvihill, E.A. (1986), Numerical simulation of the effects of varying ice crystal nucleation rates and aggregation processes on orographic snowfall. J. Clim. Appl. Meteor. 25.
3. Marshall, J.S., and Palmer, W. McK. (1948) The distribution of raindrops with size, J. Meteorol. 5:165-166.
4. Marshall, J.S., and Gunn, K.L.S. (1952) Measurement of snow parameters by radar, J. Meteorol. 9:322.
5. Imai, I., Fujiwara, M., Ichimura, I., and Toyama, Y. (1955) Radar reflectivity of falling snow, Pap. in Meteorol. and Geophys. (Japan) 6:130-139.
6. Gunn, K.L.S., and Marshall, J.S. (1958) The distribution with size of aggregate snowflakes, J. Meteorol. 15:452 (479).
7. Ohtake, T., and Henmi, T. (1970) Radar reflectivity of aggregated snowflakes, Preprints of papers presented at the 14th Radar Meteorology Conference, Tucson, Arizona, 17-20 November 1970, pp 209-211.

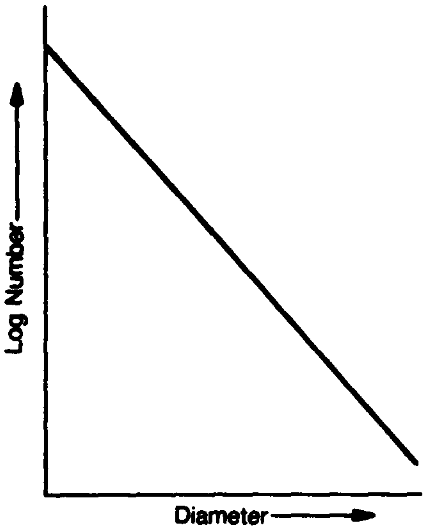

Figure 1a. A Typical Exponential Number Distribution Plotted in a Semi-Logarithmic Format

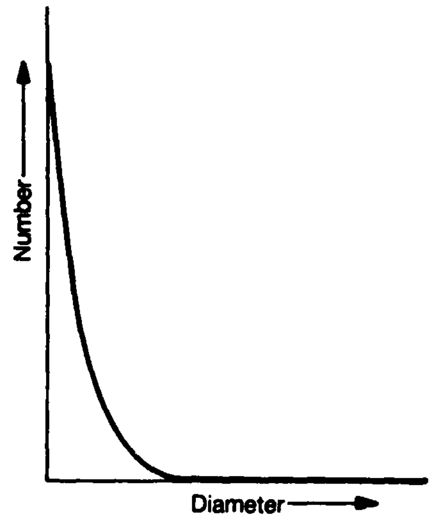

Figure 1b. A Typical Exponential Number Distribution Plotted in a Linear Format

Constant values for the slopes of exponential distribution lines are not realistic assumptions, as considerable variations in slope values are evident even in measurements of the same hydrometeor type. One particular rain situation, for example, has shown slope values that range between 1.06 and 2.2 mm^{-1} for distributions averaged over a $\sim 1\ m^3$ volume.[8] More pronounced differences in number density slopes occur with changes of hydrometeor type (rain, snow, ice crystals, etc.) with rain generally having the smallest values and the high-altitude ice crystals the largest. A considerable overlapping of values is apparent between the various types and mixtures of types, preventing specific distribution slopes being assigned to particular hydrometeors.

The study mentioned above also shows number totals of precipitable sized drops ranging from 227 to 1239 for the same averaging period. Measurements such as these point to substantial variations in both the slopes of the distribution lines and the zero intercept of the number density plots, as both these parameters have an effect on the total number of hydrometeors.

The amount of water contained in a distribution and the factor of radar microwave energy reflected from the distributed hydrometeors may be calculated from an assumed size distribution. Figure 2a shows the distributed water content and Figure 2b, the

8. Berthel, R.O., and Plank, V.G. (1983) A model for the estimation of rain distributions, Environmental Research Papers, No. 822, AFGL-TR-83-0030, AD A130080, 48 pp.

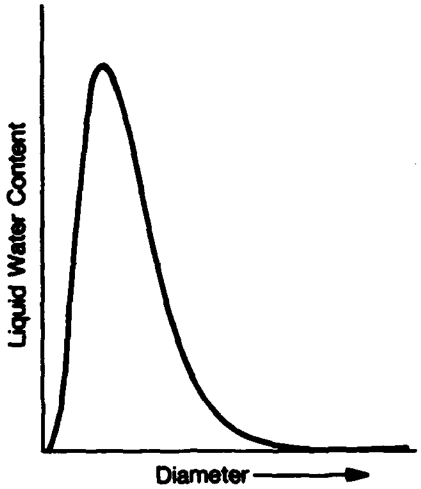

Figure 2a. A Plot of the Distributed Liquid Water Content from the Number Density Distribution of Figure 1

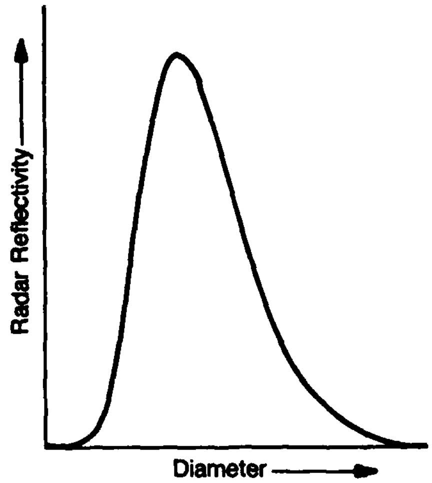

Figure 2b. A Plot of the Distributed Radar Reflectivity Factor from the Number Density Distribution of Figure 1

distributed radar reflectivity factor that would result from a full exponential distribution of rain drops as in Figure 1. The plots indicate that not only do these parameters peak at different drop sizes but that the relative water contribution (amount at a specific diameter as compared to the total) is greater than that from the relative radar reflectivity factor at the smaller sizes, and the opposite effect is prevalent at the larger diameters. Both parameters show minor relative contributions at the very small diameters with more substantial increases from the lesser numbers of larger-sized drops.

In this report, an exponential distribution refers primarily to precipitation-sized hydrometeors, as the small cloud and aerosol particles may form separate distributions of varying shapes and numerical concentrations depending upon atmospheric conditions (Section 5). The separation of cloud and precipitation hydrometeors is normally considered to occur between .05 and .1 mm in drop diameters. Because of the exponential increase in number with decreasing diameter, the total number (per unit volume) of precipitable particles is highly dependent upon this lower truncation limit, although the sensitivity of total water content or radar reflectivity to this parameter is small. On the other hand, truncating or terminating exponential distributions at the larger diameters has very little effect on total number but makes considerable differences in the total water content and radar reflectivity factor.

The following sections describe a doubly-truncated, hydrometeor model[8] of exponential type that allows variability in all the distribution parameters with only the shape of the number density curve being assumed. This hydrometeor model is then combined with empirically derived relationships to tailor distributions to particular atmospheric situations.

2. EXPONENTIAL DISTRIBUTION EQUATIONS

It has been demonstrated by several authors that the size distribution properties of raindrops, snowflakes, and ice crystals can be reasonably described by a distribution function of exponential type.[3-7] This distribution function specifies that the number concentration of the hydrometeor particles will decrease with increasing diameter (or equivalent-melted diameter) in the manner

$$N = N_0 e^{-D\Lambda} \text{ No. m}^{-3} \text{ mm}^{-1} \qquad (1)$$

where N_0 is the number per cubic meter per millimeter bandwidth at the zero intercept of the semi-logarithmic, number-density plot, Λ is the slope of the exponential number-density distribution, and D is the drop diameter in millimeters.

In the present study Eq. (1) is presumed to apply only between the truncation limits D = d (a minimum diameter) and $D = D_m$ (a maximum diameter). The double truncation of an exponential distribution function has been previously discussed by Sekhon and Srivastava.[9]

2.1 Number Total

The total number of hydrometeors (N_T) in a population described by Eq. (1) is

$$N_T = \int_d^{D_m} N \, dD \quad \text{No. m}^{-3} \qquad (2)$$

or

$$N_T = \frac{N_0 \, r_N}{\Lambda} \qquad \text{No. m}^{-3} \qquad (3)$$

9. Sekhon, R.S., and Srivastava, R.C. (1970) Snow size spectra and radar reflectivity, J. Atmos. Sci. 27:299-307.

where r_N is a "truncation ratio" specified by

$$r_N = \frac{\int_d^{D_m} N \, dD}{\int_0^{\infty} N \, dD} \quad . \tag{4}$$

or

$$r_N = e^{-d\Lambda} - e^{-D_m \Lambda} \quad . \tag{5}$$

2.2 Liquid-Water Content

The liquid-water content or mass (M) of the hydrometeor populations per unit volume is distributed with diameter as

$$M_D = \frac{\pi}{6} \times 10^{-3} \rho_{\bar{w}} N_0 D^3 e^{-D\Lambda} \quad g\ m^{-3}\ mm^{-1} \tag{6}$$

where $\rho_{\bar{w}}$ is the density of liquid water in g cm^{-3}.

The total liquid-water content of the population is

$$M = \int_d^{D_m} M_D \, dD \qquad g\ m^{-3} \tag{7}$$

that, from Eq. (6) and integration, yields

$$M = \frac{\pi \times 10^{-3} \rho_w N_0 \Gamma(4) r_M}{6\Lambda^4} \quad g\ m^{-3} \tag{8}$$

where $\Gamma(4)$ is the gamma function of 4 and r_M is a truncation ratio for liquid-water content given by

$$r_M = \frac{\int_d^{D_m} M_D \, dD}{\int_0^{\infty} M_D \, dD} \tag{9}$$

or

$$r_M = \frac{1}{6} \Big\{ e^{-d\bar{\Lambda}} \; [(d\Lambda)^3 + 3(d\Lambda)^2 + 6\, d\Lambda + 6] \; -$$

$$e^{-D_m\Lambda} \; [(D_m\Lambda)^3 + 3(D_m\Lambda)^2 + 6D_m\Lambda + 6] \Big\} . \quad (10)$$

2.3 Radar Reflectivity Factor

The distributed values of the equivalent radar reflectivity factor (Z) for the hydrometeor populations are specified by

$$Z_D = N_0\, D^6\, e^{-D\bar{\Lambda}} \; mm^6\, m^{-3}\, mm^{-1} . \quad (11)$$

The total value of the radar reflectivity factor, for the entire population of hydrometeors, is

$$Z = \int_d^{D_m} Z_D \, dD \qquad mm^6\, m^{-3} \quad (12)$$

or, from Eq. (11) on integration,

$$Z = \frac{N_0\, \Gamma(7)\, r_Z}{\Lambda^7} \qquad mm^6\, m^{-3} \quad (13)$$

where Γ (7) is the gamma function of 7 and r_Z is the truncation ratio for the radar reflectivity factor as defined by

$$r_Z = \frac{\int_d^{D_m} Z_D \, dD}{\int_0^{\infty} Z_D \, dD} \quad (14)$$

that becomes

$$r_Z = \frac{1}{720}\left\{e^{-d\Lambda}\ [(d\Lambda)^6 + 6(d\Lambda)^5 + 30(d\Lambda)^4 + \right.$$

$$120(d\Lambda)^3 + 360(d\Lambda)^2 + 720d\Lambda + 720]$$

$$- e^{-D_m\Lambda}\ \left[(D_m\Lambda)^6 + 6(D_m\Lambda)^5 + 30(D_m\Lambda)^4 + \right.$$

$$\left. 120(D_m\Lambda)^3 + 360(D_m\Lambda)^2 + 720(D_m\Lambda) + 720] \right\} . \tag{15}$$

2.4 Modal Diameters

The "modal diameters" of the M_D and Z_D distributions are characteristic parameters of the hydrometeor populations. These diameters, which specify the maximum value points of liquid-water-content and radar-reflectivity factor, are given respectively by

$$D'_M = 3/\Lambda \quad \text{mm} \tag{16}$$

and

$$D'_Z = 6/\Lambda \quad \text{mm.} \tag{17}$$

2.5 Median Volume Diameter

An additional characteristic parameter of the M_D distribution which is conventionally used and referenced is the median volume diameter (D_0). This diameter satisfies the integral relation

$$\int_d^{D_0} M_D\, dD = \int_{D_0}^{D_m} M_D\, dD \tag{18}$$

that, when the integration is performed using Eq. (6), and if all Λ terms are included on the right, yields

$$D_0 = \frac{1}{\Lambda} \ln \left\{ \frac{2[(D_0\Lambda)^3 +}{e^{-d\Lambda}[(d\Lambda)^3 + 3(d\Lambda)^2 + 6d\Lambda + 6] +} \right.$$

$$\left. \frac{3(D_0\Lambda)^2 + 6D_0\Lambda + 6]}{e^{-D_m\Lambda}[(D_m\Lambda)^3 + 3(D_m\Lambda)^2 + 6\,D_m\Lambda + 6]} \right\} \quad \text{mm.} \tag{19}$$

It is seen that D_0 is not separable in this equation. However, the equation can be readily solved by "trial and error" iteration once information about d, D_m and Λ is available.

2.6 Distribution Slope

The parameter N_0 may be eliminated between Eqs. (8) and (13) to provide an expression for the "exponential slope" of the distribution function of Eq. (1) as

$$\Lambda = 61.2 \left(\frac{M\, r_Z}{Z\, r_M} \right)^{1/3} \quad \text{mm}^{-1}. \tag{20}$$

When d and D_m are known, in Eqs. (10) and (15), this equation may also be solved by iteration.

2 7 Coefficient of the Exponential Distribution Function

N_0 may then be determined through Eqs. (8) or (13) as

$$N_0 = \frac{M\,\Lambda^4}{\pi \times 10^{-3}\, r_M} \qquad \text{No. m}^{-3}\ \text{mm}^{-1} \tag{21}$$

or

$$N_0 = \frac{Z\,\Lambda^7}{720\, r_Z} \qquad \text{No. m}^{-3}\ \text{mm}^{-1} \tag{22}$$

using $\rho_w = 1$ g cm^{-3}, $\Gamma(4) = 6$ and $\Gamma(7) = 720$.

Equations (1) through (22) constitute a descriptive equation set, or model, that may be used to determine the parameters of hydrometeor distributions when suitable input information is available.

3. EMPIRICAL RELATIONSHIPS

The initiation of any hydrometeor microphysical modeling requires input information to quantify the total number of particles and to characterize the manner that the particles are distributed, such as in the form and limits of the distribution. The 3-D model provides the liquid/ice water "content" or density, M (Eq. (8)), and ambient temperature (T°C) of hydrometeors contained within a specific spatial volume.* M is an indirect measure of number density since it is dependent upon both the number and sizes of the hydrometeors being considered. Utilization of the hydrometeor model specifies an exponential form of particle distribution; however, any given M may be represented by a multitude of exponential distributions with various slopes and numbers of particles.

With a second measure of a particular distribution, such as Z, one may gain knowledge of Λ through the distributed values of M and Z, since M is a function of the drop diameters to the third power (Eq. (6)) while Z is a function of the diameters to the sixth power (Eq. (11)). Even with this additional information, the total number of hydrometeors may vary considerably depending upon the upper and lower truncation limits (d and D_m) of the distribution.

Therefore, since the 3-D model gives only the M and T, additional parameterization (Z, d and D_m) must be specified or inferred from empirical relationships to supply the necessary information to utilize the equation set of Section 2.

For any single distribution, M and Z can be expressed as a proportionality factor by

$$k = M\ Z^{-0.5} \qquad g\ mm^{-3}\ m^{-1.5}. \tag{23}$$

This k factor will vary depending upon the number and size distribution of the drops contained within the diameter limits of d to D_m. Although k is determined from calculations of M and Z that are functions of drop sizes, the concept may be utilized with snow/ice hydrometeors since k is relatively insensitive as to the method used to deduce equivalent melted diameters from the frozen particles.[10]

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*In many cloud models that predict mixing ratios (r), M is often calculated as the product $r\rho_o$, where ρ_o is the dry air density.

10. Plank, V.G., Berthel, R.O., and Barnes, A.A., Jr. (1980) An improved method for obtaining water content values of ice hydrometers from aircraft and radar data, J. Appl. Meteor. 19:1293-1299, AFGL-TR-81-0011, AD A094328.

This relationship is valid for any single hydrometeor distribution no matter what shape or form since k can also be written as

$$k = C\,F\,N_T^{0.5} \qquad g\ mm^{-3}\ m^{-1.5} \tag{24}$$

where

$$C = \frac{\pi}{6} \times 10^{-3} \tag{25}$$

and F is a "form factor" that describes drop diameter apportionment relative to each other and to the total number concentration of the complete distribution. These expressions relating k and N_T have been described and discussed elsewhere.[10,11]

Past investigations[12] have shown that k varies with T as

$$k = .0122\ e^{-.0474\ T} \qquad g\ mm^{-3}\ m^{-1.5} \tag{26}$$

for data taken over Wallops Island, VA and

$$k = .0143\ e^{-.0291\ T} \qquad g\ mm^{-3}\ m^{-1.5} \tag{27}$$

for data acquired over the Kwajalein Atolls in the Pacific for T between −70 and 20°C.

Equations (26) and (27) are derived from all data available in the data sets and considerable scatter is evident since they include individual aircraft samples of varying time durations in different weather situations. These variabilities affect the k factors as (1) the shape of the hydrometeor distribution becomes more exponential with longer averaging time,[13] and (2) storms experiencing atmospheric turbulence produce more mixing of hydrometeor types at comparable temperature levels. No compensation

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11. Plank, V.G. (1977) Hydrometeor data and analytical-theoretical investigations pertaining to the SAMS Missile Flights of the 1972-73 season at Wallops Island, Virginia, Environmental Research Papers, No. 603, AFGL/SAMS Report No. 5, AFGL-TR-77-0149, AD A051192, 239 pp.

12. Dyer, R.M., Berthel, R.O., and Izumi, Y. (1981) Techniques for measuring liquid water content along a trajectory, Environmental Research Papers, No. 733, AFGL-TR-81-0082, AD A102922, 52 pp.

13. Plank, V.G., Berthel, R.O., and L.V. Delgado (1980) The shape of raindrop spectra for different situations and averaging periods, J. Rech. Atmos. 14:301-309, AFGL-TR-81-0008, AD A094877.

can be made for the varying time durations of the original data, but the selection of samples in accordance with storm type serves to mitigate the effects of different weather situations. Since the present usage of the 3-D model is for convective storms, the k data were reanalyzed to form an equation tailored to this type.

The Wallops data, consisting mainly of stratified storms and cloud situations, were disregarded. The Kwajalein data, on the other hand, are all from convective cases. However, these data include one particularly intense storm that produced a k-T relationship

$$k = .0174\, e^{-.042\, T} \qquad g\, mm^{-3}\, m^{-1.5} \tag{28}$$

that is quite different from the more normal situations (Figure 3). The remaining storms gave

$$k = .00963\, e^{-.031\, T} \qquad g\, mm^{-3}\, m^{-1.5} \tag{29}$$

thus Eq. (29) was chosen for use in the 3-D model analysis.

When M and T are known, Eq. (29) may be substituted into Eq. (23) and solved for Z to determine the equivalent-radar reflectivity for a particular hydrometeor distribution as

$$Z = \left(\frac{M}{.00963\, e^{-.031\, T}}\right)^2 \qquad mm^6\, m^{-3}. \tag{30}$$

Temperature may also be associated with the upper truncation limit as it can be indirectly linked to a combination of D_m and Λ, as used in Eqs. (10) and (15), when considering an exponential form for number density.

The combined parameter, $D_m\, \Lambda$, has previously been related to hydrometeor type[11] with a value of 8.2 determined for widespread rain[14], 9 for large snow[15] and the approximate values of 10.5 and 12 assigned to small snow and ice crystals (bullet

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14. Plank, V.G., and Berthel, R.O. (1982) A descriptive double-truncated exponential model for hydrometeors of precipitable size, Conf. on Cloud Physics, Chicago, Nov 15-18:190-194, AFGL-TR-82-0347, AD A122036.

15. Berthel, R.O. (1980) A method to predict the parameters of a full spectral distribution from instrumentally truncated data. Environmental Research Papers, No. 689, AFGL-TR-80-0001, AD A085950, 48 pp.

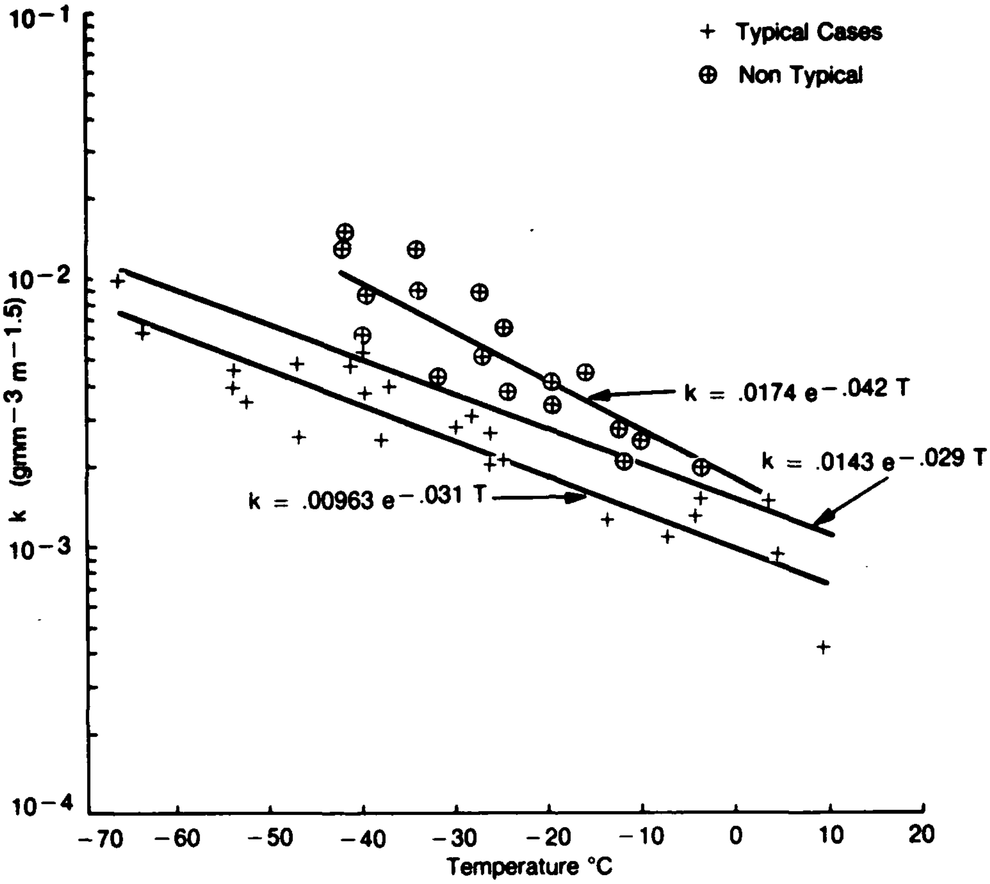

Figure 3. Aircraft Acquired k Factors and Temperatures from the Kwajalein Data

rosettes). When the averaged T's from the aircraft measurements of the typical Kwajalein storms are associated with the $D_m\Lambda$'s of the various hydrometeor regions, they generally conform to a relationship described by

$$D_m\Lambda = 8.2\ e^{-.00846\ T} \qquad <0°C \tag{31}$$

with

$$D_m\Lambda = 8.2 \qquad \geq 0°C. \tag{32}$$

It is assumed that Eqs. (31 and 32) are valid up to the raindrop break-up size of ~6 mm. At the point where D_m reaches 6 mm, Λ equals 1.367 mm^{-1} and corresponds to a particular mass, M_B. Since k continues to decrease with increasing T, most likely reflecting evaporation and coalescence processes, M_B is dependent upon T and can be calculated using Eq.'s (8), (10) and (29) (the value used for d is relatively unimportant in these calculations of large M and Z's). The M at the point where D_m reaches break-up size forms a direct relationship with T as

$$M_B = 6.144\, e^{-.062\, T} \quad g\, m^{-3} \quad . \tag{33}$$

As M increases in excess of M_B with D_m held constant at 6 mm, the definition of $D_m\Lambda$ in Eq. (31) and (32) is no longer valid and is replaced by

$$D_m\Lambda = 6 \tag{34}$$

in the calculations of r_M (Eq. (10)) and r_Z (Eq. (15)).

The Z and $D_m\Lambda$ values determined from the above relationships may now be combined with a specified lower truncation limit, d, for application to the equation set of Section 2. Calculations of Λ, N_0, N_T and D_m are shown plotted versus M in Figures 4 through 7 for various values of T and d.

4. DERIVED RELATIONSHIPS

The number and sizes of hydrometeors at some particular time and place in a storm's life cycle are important requisites for the 3-D model. Using this information, the model may then apply various physical processes to extrapolate future conditions.

Assuming that the exponential distribution equations may provide reasonably accurate determinations, the distributed number may be deduced from Eq. (1) and the total number from Eq. (3). However, both equations require prior knowledge of other distribution parameters.

The sequence that would normally be used to parameterize an exponential distribution from prior knowledge of M and T is the determination of Z (Eq. (30)), the iterative calculation of Λ (Eq. (20)), the evaluation of N_0 (Eq. (21 or 22)) and the application of these parameters to the distributed N (Eq. (1)) or total N (Eq. (3)) equations. This procedure produces very accurate values for Λ, but at a high computational expense. Although this circuitous procedure is highly complicated,

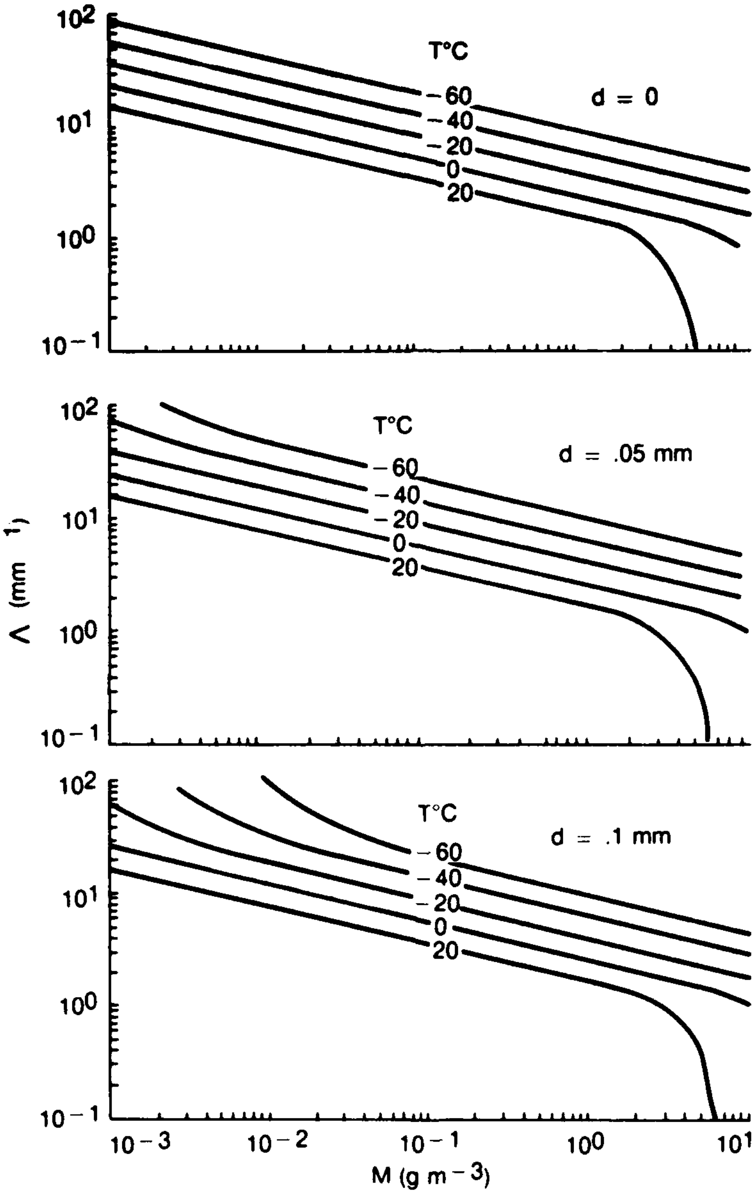

Figure 4. Plots of Λ Versus M for Values of T = -60, -40, -20, 0 and 20°C for d = .1, .05 and 0 mm. Deviations from the straight-line plots at higher M's and T's are the result of distributions being truncated at the water drop, break-up size of 6 mm

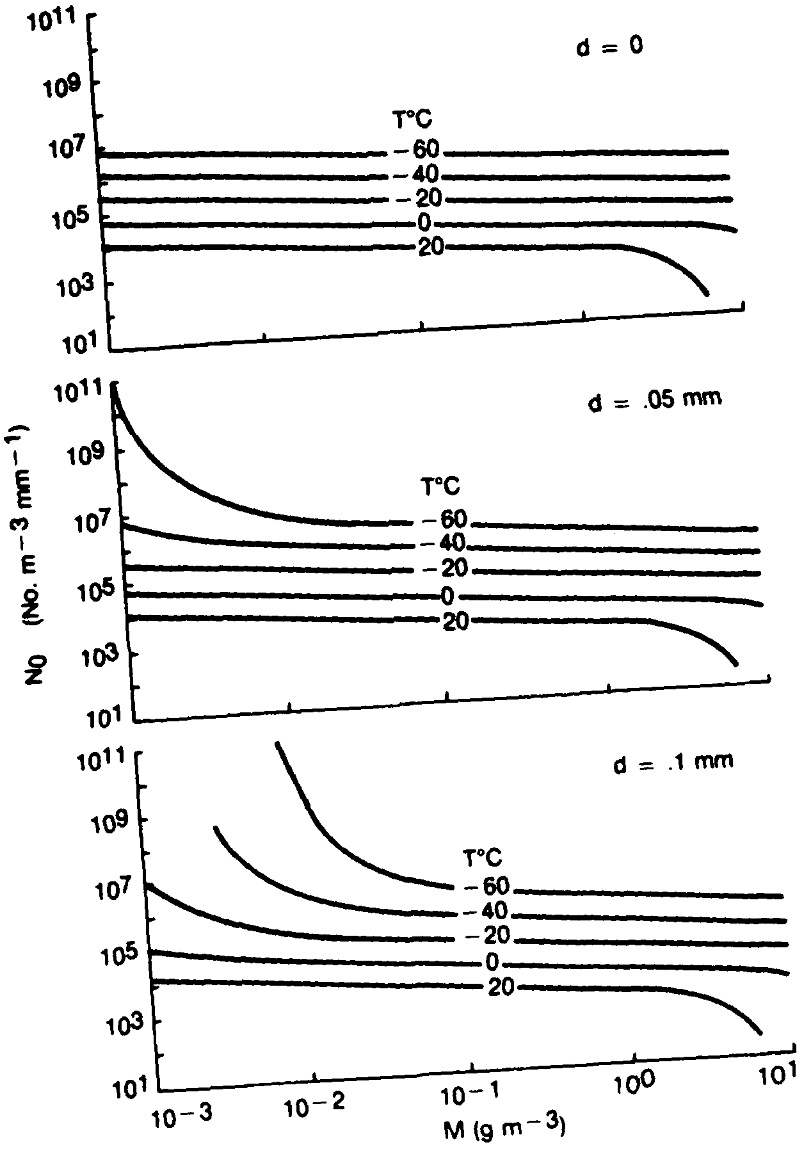

Figure 5. Plots of N_0 Versus M for Values of T = -60, -40, -20, 0 and 20°C for d = .1, .05 and 0 mm. Deviations from the straight-line plots at higher M's and T's are the result of distributions being truncated at the water drop, break-up size of 6 mm

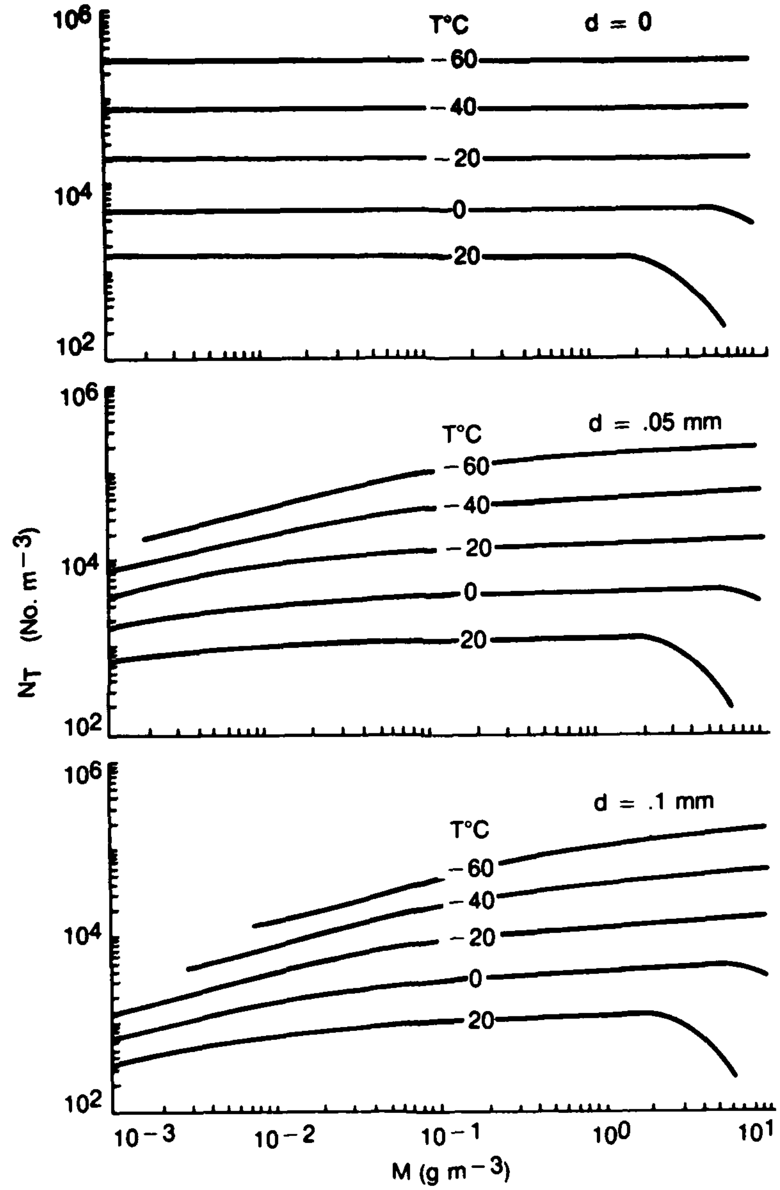

Figure 6. Plots of N_T Versus M for Values of T = -60, -40, -20, 0 and 20°C for d = .1, .05 and 0 mm. Deviations from the straight-line plots at higher M's and T's are the result of distributions being truncated at the water drop, break-up size of 6 mm

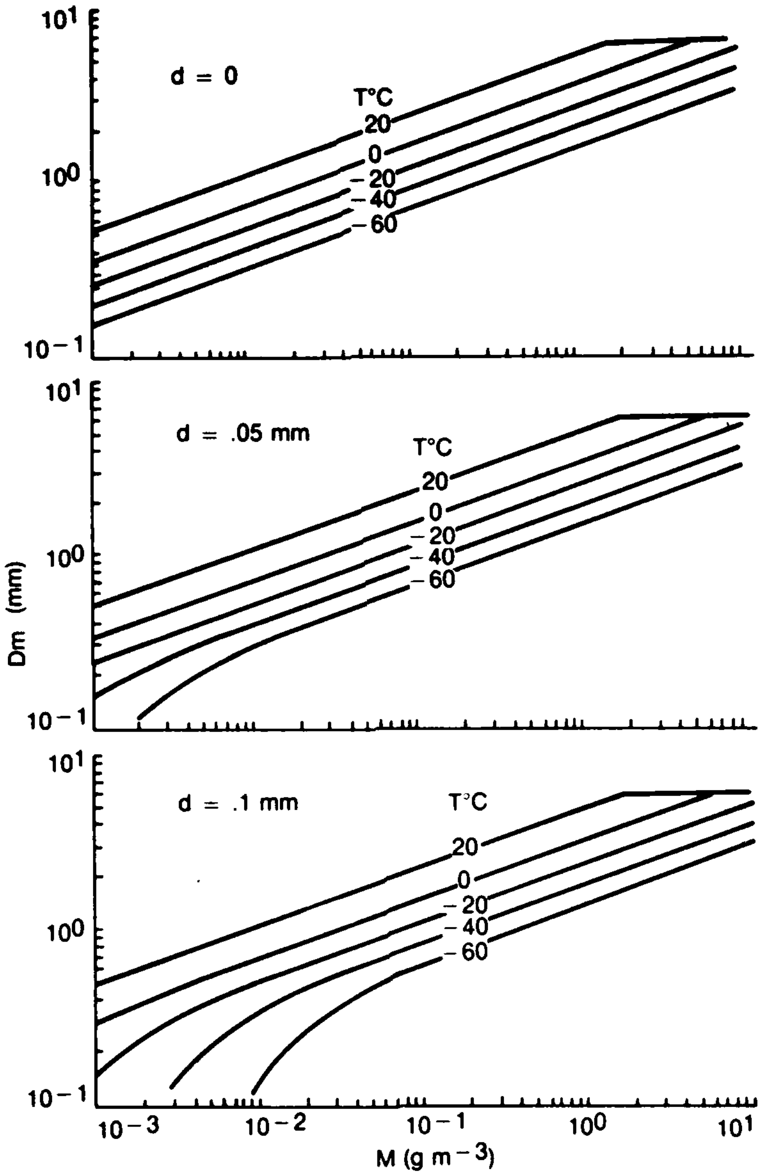

Figure 7. Plots of D_m Versus M for Values of T = -60, -40, -20, 0 and 20°C for d = .1, .05 and 0 mm. Deviations from the straight-line plots at higher M's and T's are the result of distributions being truncated at the water drop, break-up size of 6 mm

especially the iterative process, it may be easily solved by computer operation. However, when a multitude of individual distributions are required, as in the everchanging water budget in a 3-D model, much valuable computer time would be consumed. Thus, we have investigated various means of estimating Λ to eliminate the iteration.

The most straight-forward method considered was the determination of Λ by assuming d = 0 as a first-order estimate in the calculations of r_M and r_Z in Eq. (20) as

$$r_{M,o} = 1 - \frac{e^{-D_m \Lambda}}{6} \left[(D_m \Lambda)^3 + 3(D_m \Lambda)^2 + 6D_m \Lambda + 6 \right] \tag{35}$$

and

$$r_{Z,o} = 1 - \frac{e^{-D_m \Lambda}}{720} \left[(D_m \Lambda)^6 + 6(D_m \Lambda)^5 + 30(D_m \Lambda)^4 + 120(D_m \Lambda)^3 + 360(D_m \Lambda)^2 + 720(D_m \Lambda) + 720 \right] . \tag{36}$$

We can then rewrite (20) and apply (23) to obtain:

$$\boxed{\Lambda_E = 61.2 \left(\frac{k^2 \, r_{Z,o}}{M \, r_{M,o}} \right)^{1/3} \quad mm^{-1}.} \tag{37}$$

Since the product $D_m \Lambda$ is known from Eq.'s (31), (32), or (34) and k is known from Eq. (29), both as functions of temperature, evaluation of Λ_E using this expression is very simple.

We then evaluated the assumption d = 0 by comparing values of Λ's obtained by the simple solution with those obtained by the more accurate iterative method.

When the Λ's estimated from the simple method (called Λ_E) are compared with those calculated by iteration using a specified nonzero lower truncation limit (referred to as Λ), they agree favorably at higher values of M (> 0.1 g m^{-3}) and also at the higher temperatures as shown in the plots for d = .05 and .1 mm in Figure 8. The agreement becomes noticeably better as the specified value for d becomes smaller, and when d = 0 the ratio of Λ_E / Λ becomes unity. The discrepancies between Λ_E and Λ for $d > 0$ are greater at larger d values, smaller M's and lower temperatures.

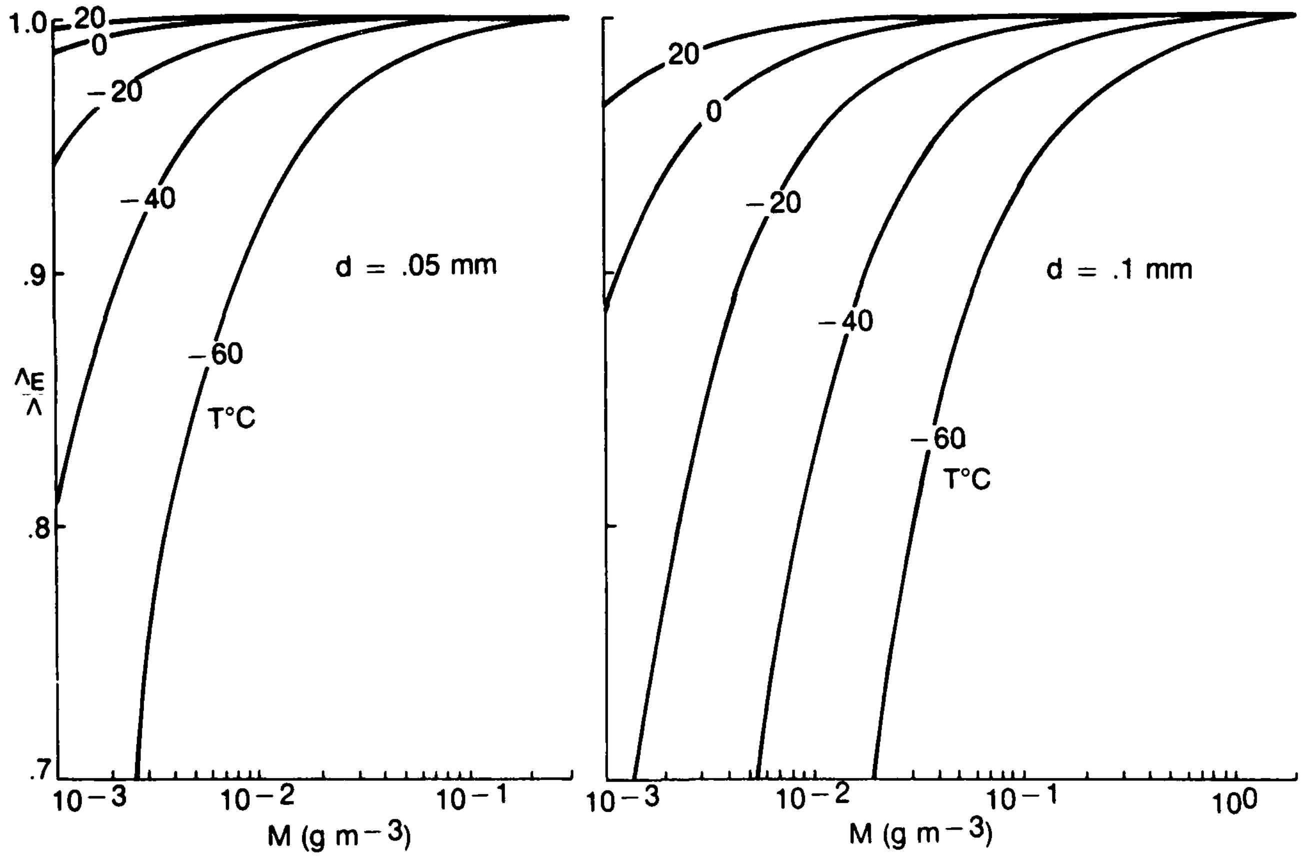

Figure 8. Plots showing the Factors of Estimated Λ's from the d = 0 Method Divided by Those Calculated from the Equation Set of Section 2 for d = .05 and .1 mm and T's of -60, -40, -20, 0 and 20°C

The estimates provided by assuming $d = 0$ are, in most cases, good for situations that may be classified as being normal. For example, if one were to consider a situation where $M = .01\ g\ m^{-3}$ at $-40°C$, Λ_E would equal $28.8\ mm^{-1}$. If under these conditions the lower truncation limit were designated as being .1 mm, Λ_E would be a factor of .835 of the Λ calculated by iteration ($34.5\ mm^{-1}$). However, at $-40°C$ one would expect particle distributions to have lower values of d, such as .04 mm, where Λ_E now becomes a factor of .99 of the calculated Λ of $29.1\ mm^{-1}$. Thus, this method is adequate for the estimation of Λ when considering a normal situation. Non-normal cases (i.e. cases with large d, small M, and low T) will either be required to revert to the trial-and-error solution or to use to some other correction procedure.

Figure 9 shows the M and T values that define a factor (f) of .99 from Λ_E/Λ for various values of d. Any combination of M and T that fall above the line of designated d will give a Λ_E within 1% of the calculated value. Similar relationships as shown in Figure 9 can be derived for other factors and may be expressed in terms of M, T and d as

$$f = \frac{\left(\dfrac{M}{d^3\ e^{-.066\ T}}\right)^{-.94} - 8.26}{-8.22} . \tag{38}$$

When $f \geq .5$, Eq. (38) may thus be used to adjust a Λ_E that was determined by assuming $d = 0$ by

$$\Lambda = \Lambda_E / f \tag{39}$$

to produce a Λ that will agree within 1% to one calculated from the equation set of Section 2. Note that the use of Eq. (38) allows d to be determined independently. For example, smaller values of d may be used at altitudes where clouds are present, with larger values being employed below cloud base to represent the effects of evaporation.

The values of N_0 may then be found by applying the corrected Λ_E to Eqs. (10) and (21) and to Eqs. (3) and (5) for the determination of N_T.

The $d = 0$ method for estimating Λ is only applicable to distributions where D_m is smaller than the break-up size of rain drops ($D_m < 6$ mm) since D_m is a constant for M quantities above M_B (Eq. (33)). However, estimations of Λ, when $D_m = 6$ mm, may be found by the equations

$$\Lambda_E = 2.293\ e^{-.084\ M\ e^{.062\ T}} \quad mm^{-1} \tag{40}$$

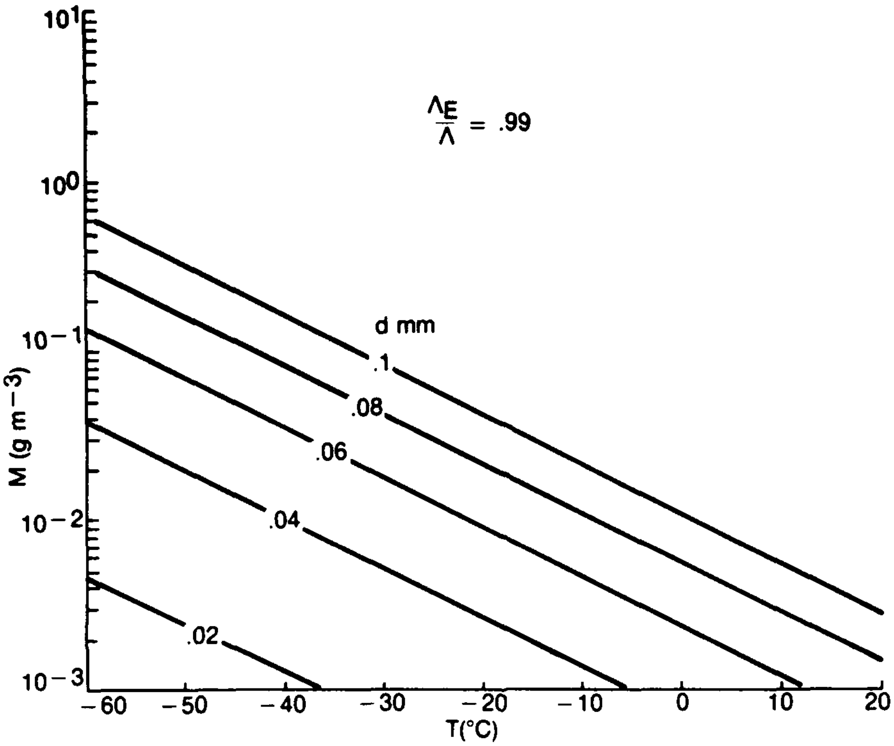

Figure 9. The M and T Values that Define a Factor (Λ_E/Λ) of .99 for d = .02, .04, .06, .08 and .1 mm

for M values less than

$$M = 11\, e^{-.062\,T} \qquad g\ m^{-3} \tag{41}$$

and

$$\Lambda_E = 1.793 - .0802\, M\, e^{.062\,T} \qquad mm^{-1} \tag{42}$$

when M is equal to or larger than Eq. (41). The Λ_E's determined from Eqs. (40) and (42) are within 1% of the calculated values.

5. REMARKS

The concept of hydrometeor modeling presented in this report involves a blending of theoretical and empirical relationships. It should be considered as a starting point or corner stone for the building of realistic hydrometeor distribution predictions from large-scale meteorological models such as the AFGL 3-D model. Several aspects of this modeling need to be explored further.

Distribution shape has been investigated many times in a variety of ways. The large or precipitation-sized drops generally show a decrease in number with increasing size and, in most cases, can be reasonably described by an exponential curve. However, there are distributions that do not conform such as those displaying bi-modal tendencies. Those non-conformists can possibly be associated with particular cloud or storm situations for future modeling efforts.

Also, the exponential number density is usually associated with the precipitation-sized particles. Since cloud hydrometeors are heavily dependent upon cloud type, the shape of the number-density curve in the small diameter range changes with different cloud situations. However, it has been pointed out[16] that a slight alteration by the inclusion of D^u and K in the distribution function of Eq. (1) to form a gamma distribution as

$$N = K\, N_0\, D^u\, e^{-D\Lambda} \qquad \text{No. } m^{-3}\, mm^{-1}, \tag{43}$$

where $K = 1/mm\,\mu$, can conveniently change the curve shape especially in the small size range. Figure 10 shows the varying shapes that result from different values of u in Eq. (43) with u = 0 being an exponential distribution. Again, association of different cloud and storm situations with this type function may result in more realistic distribution predictions.

The relationships of T with $D_m\Lambda$ and the k factor have been investigated for two locations, Wallops Island and Kwajalein, and deserve further study. Associations of hydrometeor type with both $D_m\Lambda$ and k have been employed in the past with varying amounts of success but the concept of relating these parameters with T has only

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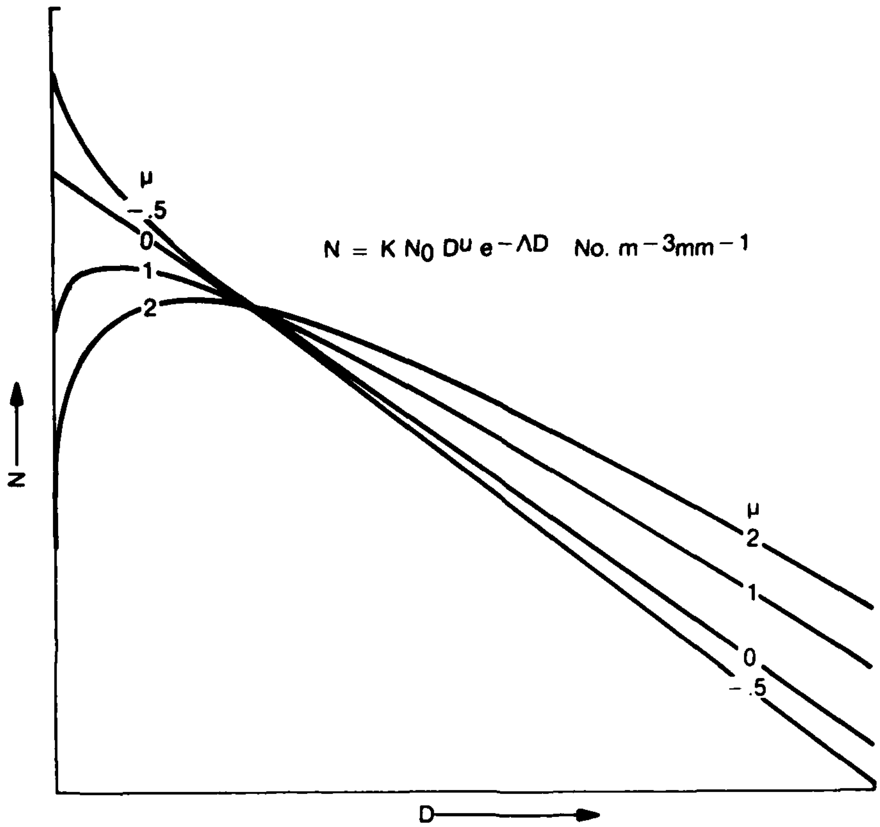

Figure 10. Changes in the Number Density Curve from Different Values of u

recently been a serious consideration. A reanalysis of past aircraft-acquired, PMS 1-I data[17] is currently underway at AFGL in an attempt to verify or tighten these relation ships and/or to derive equations for application to different weather situations.

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